Performance Based Learning and Assessment Task

Your School's Athletic Director Needs Your Mathematical Input!

I. **ASSESSSMENT TASK OVERVIEW & PURPOSE:**
Students will be using a variety of geometrical concepts and skills as they work with a partner to determine the most cost effective way of covering a rectangular football field with artificial grass. Students must develop and implement a plan of action so that they will be able to come up with a solution to present.

II. **UNIT AUTHOR:**
Yolonda Shields, Benjamin Franklin Middle School, Franklin County Public Schools

III. **COURSE:**
Geometry

IV. **CONTENT STRAND:**
Measurement & Geometry

V. **OBJECTIVES:**
SOLs: G.10, G.3, G.14, G.8

VI. **REFERENCE/RESOURCE MATERIALS:**
Students will use: TI-83 Plus (or higher) Graphing Calculator, Computer, Pencil, Paper, Ruler/meter stick, Construction Paper, Internet, Protractor, Geometer's Sketchpad Software (optional), Assessment Rubric, Copy of Performance Task, Word Processing Software (i.e Microsoft Word or Google Docs)

VII. **PRIMARY ASSESSMENT STRATEGIES:**
Rubric/list for teacher and student, assessment list for each of the activities (includes the mathematics content, process skills, and requirements for the finished product)

VIII. **EVALUATION CRITERIA:**
Students will be evaluated on their completion of the activity and by their written letter to the school's athletic director

IX. **INSTRUCTIONAL TIME:**
Two ninety-minute class sessions

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Strand

Measurement & Geometry

Mathematical Objective(s)

Students will be able to: 1)T.S.W solve real-world problems involving angles of polygons, 2)T.S.W identify tessellations in construction and nature, 3)T.S.W use similar geometric objects in two-dimensions to solve real-world problems about similar geometric objects (making a scale drawing to model the football field), 4)T.S.W solve real-world problems using right triangle trigonometry and properties of right triangles (finding the area of regular hexagons), 5)T.S.W determine the most cost effective method for covering the surface of a rectangular football field with artificial grass.

Related SOLs

- SOL G.3d (The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods)
- SOL G.8 (The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry)
- SOL G.10 (The student will solve real-world problems involving angles of polygons)
- SOL G.14 (The student will use similar geometric objects in two- or three-dimensions to d) solve real-world problems about similar geometric objects)

NCTM Standards

- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others
- Draw and construct representations of two-dimensional geometric objects using a variety of tools.
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest.
- Make decisions about units and scales that are appropriate.
- Understand and use formulas for the area of geometric figures.
- Organize their mathematical thinking through discussion with peers.

- Communicate their thinking clearly to teacher and peers.
- Analyze and evaluate the mathematical thinking and strategies of their partners.
- Use the language of mathematics to express mathematical ideas precisely.
- Create and use representations to record and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations.
- Use representations to model and interpret physical and mathematical phenomena.

Materials/Resources

Students will need: TI-83 Plus (or higher) Graphing Calculator, Computer, Pencil, Paper, scissors, Ruler/meter stick, Construction or White Copy Paper, Internet, Protractor, Geometer's Sketchpad Software or GeoGebra(optional), Assessment Rubric, Word Processing Software (i.e Microsoft Word or Google Docs)

Assumption of Prior Knowledge

- Definition of tessellation and ability to connect this to "covering the plane/football field."
- Knowledge of which polygons will tessellate the plane using knowledge of interior angles
- How to determine the total cost of an item based on given information
- Finding the area of regular polygons (including hexagons using right triangle trigonometry)
- Using Word Processing Software
- Making Scale Drawings and using proportional reasoning
- Some experience with graphing calculator technology and Geometer's Sketchpad / GeoGebra Software (optional)

Introduction: Setting Up the Mathematical Task

The mathematical goal of this activity is for students to choose between a square and a hexagon shape to use to cover the surface of a football field with artificial grass. The artificial grass comes in "cylindrical rolls." Each square roll is made up of 5 foot squares (5 ft. x 5 ft.) that are 2 squares wide and 10 squares long. Each hexagonal roll is made up of regular hexagons with sides that are also 5 ft. each. The widest part of the cylindrical roll is 2 hexagons wide and 10 hexagons long. Students will have to be able to determine how many rolls of each type would be needed to cover the rectangular surface, which would help them determine the total cost for using squares versus using hexagons. In doing so, students have to verify that these two options do tessellate the plane. Given the cost of both the square roll and the hexagonal roll, the students would then have to determine the most cost effective shape and prove their point to the athletic director at their school. The goal is also to help the students help the school save money and make an informed decision using the mathematics to back them up. Students will use word processing software and scale drawings to show the athletic director why he or she should use their particular shape.

Teachers should set-up the task as follows:

- Discuss the activities for the day (refer to displayed Agenda)
- Inform students that the school is going to be resurfacing the football field with artificial grass.
- Brainstorm/Discussion (Think- Pair Share then Whole Group Discussion): What are the benefits of artificial grass versus real grass?
- Motivating Activity: Inform students that the athletic director wants to gather information that could help the school save money for this project and that the director is asking our class to help him by using our knowledge of Geometry and other math concepts/skills to determine what to do given 2 different options/shapes to use to cover the surface. The students will be typing a letter to the athletic director and give him solid mathematical proof about why their solution works best, a scale drawing of their scenario, and the cost of their particular project.
- The teacher will pass out the mathematical task, which also includes the information for the cost per each type of cylindrical roll.
- The teacher will pass out a rubric and assessment list that students will use as they work in groups of 2 as they work to come up with a solution to this real-world problem. The teacher will reinforce the idea that he or she is going to be in the role of facilitator and that the students will be responsible for developing a plan and implementing that plan to come up with a solution. Students will have to use Mathematics to solve the problem, but will be given access to a computer and the Internet to help them.

Student Exploration

Students will first need to research the dimensions of a football field from post to post. They will need to convert this to feet since each of the pieces have sides that measure 5 feet each. Students will also need to determine if each of the 2 shapes (squares and hexagons) will indeed tessellate the plane and what would be the disadvantage if we used a shape that does not tessellate the plane (i.e gaps/more waste). Students will also be determining how many rolls (which consist of these grass pieces that are pieced together 2 wide and 10 long) would be needed to cover the field and the total cost. Then, students will be doing scale drawings to model this and to include with their letter to the Athletic Director. Students will have access to the computer for research, typing their paper, and using Geometer's Sketchpad Software if desired. They will also have access to the Internet and a graphing calculator to help them with these calculations.

Student/Teacher Actions

- I expect students to work together as they communicate their thinking and their new knowledge with each other and with the teacher.
- The teacher will help students with any clarification needs and assist students who are having difficulties by helping them connect their previous experiences to these new ones.
- If there are students who are ready to move forward, differentiation will be used and the students can explore additional costs associated with covering the surface of a football field (i.e. the adhesive/tools needed to secure the grass). This information could also be included in the final product.
- Closure will involve the students sharing their letters with the other students and their being a whole group discussion of the various solutions. Students will also send their letters to the Athletic Director who will later give them feedback.

Assessment List and Benchmarks

Attached are class worksheets, rubrics, and benchmarks for the task.

Performance Based Assessment Task

Name(s): ________________________

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Date: ________________

Geometry Teacher: _________________

Your School's Athletic Director Needs Your Mathematical Input!

You and your partner are being asked for your mathematical input. Our school football field has become very expensive to maintain and has been a safety concern for the student athletes. Given the decrease in funding for the school, changes need to be made in order to save money and decrease the safety hazards. Our school's Athletic Director is considering resurfacing the field with artificial grass. He has already chosen the type of grass he wants to use on the field and knows its cost per cylindrical roll. The athletic director wants to gather information that could help the school save money for this project and the director is asking you and your partner, as members of our Geometry class, to use your knowledge of Geometry and other math concepts/skills to determine what to do given 2 different options/regular shapes (squares and hexagons) to choose from. Each of the shapes has sides that are 5 feet and come in "cylindrical rolls". Each square roll is made up of 5 foot squares (5 ft. x 5 ft.) that are 2 squares wide and 10 squares long. Each hexagonal roll is made up of regular hexagons with sides that are also 5 ft. each. The widest part of the cylindrical roll is 2 hexagons wide and it is 10 hexagons long. Each of these artificial turf rolls costs the same ($2400) and you have to determine which one will best fit the needs of our school. Afterwards, you will present your mathematical evidence in the form of a typed letter to the athletic director. This letter will need to present a solid mathematical proof about why your solution works best, a scale drawing or other type of relevant drawing of your scenario, and the cost of your particular project. If you and your partner work together well and do a great job, it might just be your solution that helps your school make the right and most cost effective decision!

Assessment List and Benchmarks

			Earned Assessment	
#	Element	Point Value	Self	Teacher
1	Student participated in discussions with his/her partner and the class by speaking or asking a question at least three times (recorded with an observation checklist by instructor).	2		
2	Student developed a plan for solving the real-world problem involving covering the football field with artificial grass.	2		
3	Student researched and found realistic dimensions for a football field from post to post.	2		
4	Student recorded measurement findings while completing the activity.	2		
5	Student correctly determined which of the 2 shapes could be most effective in this project.	2		
6	Student explained their mathematical reasoning behind their shape selection.	2		
7	Student determined how many rolls of artificial grass would be needed to cover the surface.	2		
8	Student correctly determined the cost associated with covering the football field with these rolls of artificial grass and used Mathematics to arrive at their conclusion.	2		
9	Student demonstrates use of technology in their solution and final product (letter to Athletic Director).	2		
10	Student completes a scale drawing or other relevant type of drawing as a model to show a visual of how the football field would look.	2		
11	Student uses Word Processing Software to type a letter to the Athletic Director.	2		
12	Student shares final product with the class in the form of a presentation that is at least 3 minutes long.	2		
13	Student's work is well organized.	2		
14	Student's work is neat.	2		
15	Student can explain their reasoning for their ideas, formulas, and work shown from activity.	2		

Benchmark

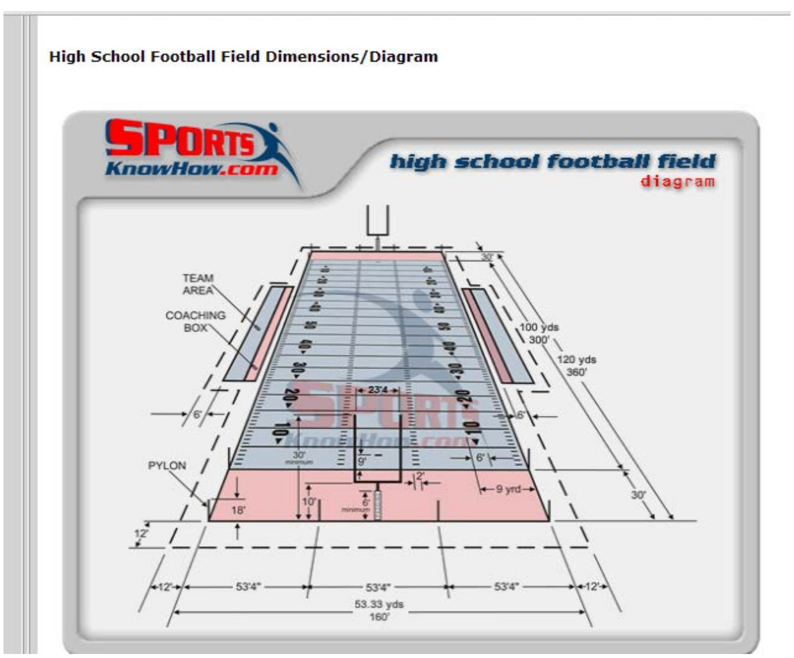
(Sample Letter to the School Athletic Director with Research, Drawings, and Mathematical Calculations Used to Make Our Decision)

4/25/14

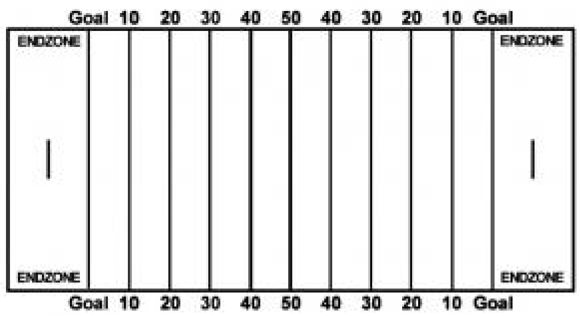
Dear Mr. Walker,

Thank you for allowing our Geometry class to be a part of the decision-making process for determining which option would be best for covering our school football field with artificial grass. We worked together in groups of 2 and were tasked with using Mathematics and research to help come up with the most cost effective solution. Our group determined that you should choose the hexagonal cylindrical rolls. By doing so, you will save our school lots of money! In fact, the cost savings would be $134,400. The information and the steps we used to reach our decision are below:

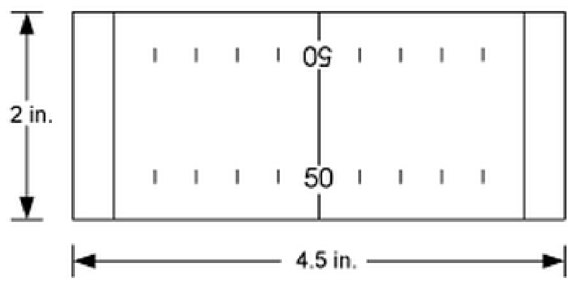
Research of High School Football Dimensions:



A football field is rectangular. The area that will be covered with the artificial grass has dimensions of 53 1/3 yards by 120 yards. After doing mathematical conversions, this means the football field is 160 feet by 360 feet. The surface of the football field is like a "plane" in mathematics since it is two-dimensional.

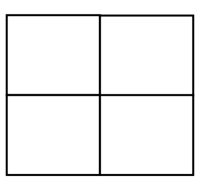


2. The scale drawing of the football field shown below is drawn using a scale of 1 inch (in.) = 80 feet (ft).

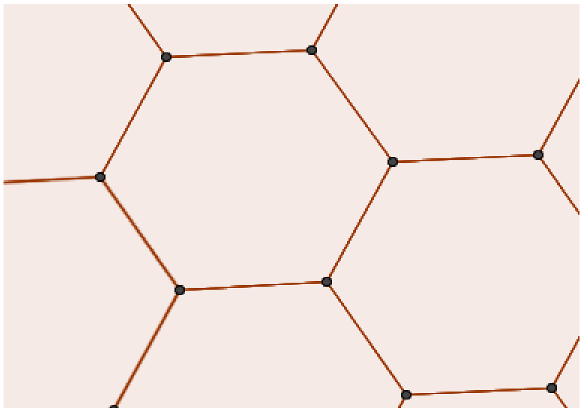


Will Squares and Hexagons Tessellate the Plane By Themselves?

In a tessellation, there are no gaps or overlaps. Squares have interior angles that are 90 degrees. Since 90 is a factor of 360, squares do tessellate the plane. Around each vertex, there would be 4 squares.

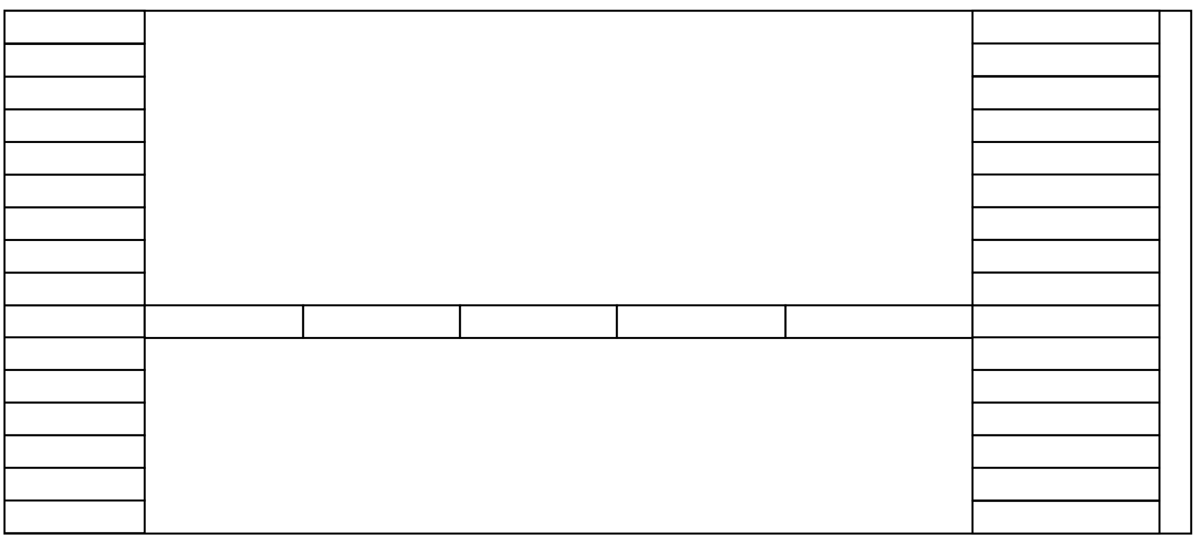


Regular hexagons have interior angles that are 120 degrees each. Since 120 is a factor of 360, they will also tessellate the plane. Around each vertex, there would be 3 regular hexagons.



How many Square Cylindrical Rolls of Artificial Grass Would Be Needed to Cover the Football Field?

Work: Each Square roll is 10 ft. by 50 feet b/c 2 * 5 = 10 and 10 * 5 = 50



160 ft.

360 ft.

* The diagram above can be filled in even more, but each of the boxes represent the amount of space each of the square cylindrical rolls will take up. To cover the width of the football field, 16 rolls would be needed across. To cover the length, 7 rolls will cover 350 feet, but there would be a 10 foot by 160 foot strip of the field that would be uncovered. Since the area of this part would be 1600 square feet

(10 * 160) and each square cylindrical roll is 500 square feet (10 * 50), 4 additional rolls would be needed. To find the total number of square cylindrical rolls, we would multiply 16 * 7 to get 112. Then, we would add 4 to get 116 total square cylindrical rolls.

How many Hexagonal Cylindrical Rolls of Artificial Grass Would Be Needed to Cover the Football Field?

<u>Student Option #1</u>

Image of cylindrical roll made out of hexagons (2 wide and 10 long):

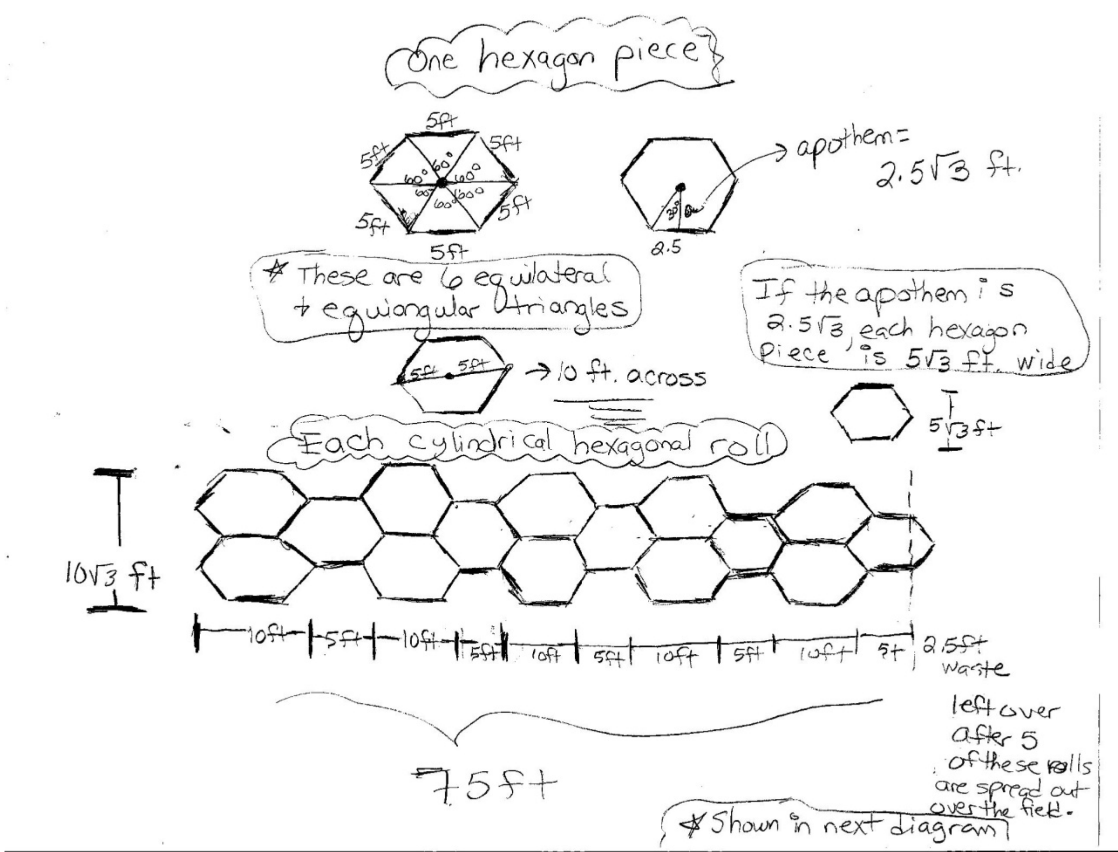
Each roll consists of 15 hexagons. The formula for the area of any regular polygon is ½ aP, where a is the apothem and P is the perimeter. The apothem for each of these hexagons is 2.5 * (square root of 3) and the perimeter is 30 (5 * 6). So, the area of each hexagon is 37.5 * (square root of 3). Since each roll contains 15 hexagons, then the total square footage would be 37.5 * (square root of 3) * 15 = 562.5 * (square root of 3) = 974.27813 square feet. Since the football field is 160 feet by 360 feet, the total square footage is 57,600 square feet. To find the number of rolls needed, we would take 57,600 and divide it by 974.27813. This gives 59.12069 rolls, which would be rounded up to 60 rolls. Since hexagons are made up of six equilateral triangles, the ends of the football field could be filled in by these shapes if needed. So, there would be little or no waste.

<u>Student Option # 2</u>

I used GeoGebra to create a model of how one cylindrical hexagonal roll would look laid out flat. I then used Microsoft Word to copy this image 21 times on one page so that I could create a template of rolls that I could cut out, manipulate, and fit onto a football field that I sketched on a 8 ½ " by 11" piece of copy paper. Here is a picture of the template I created:

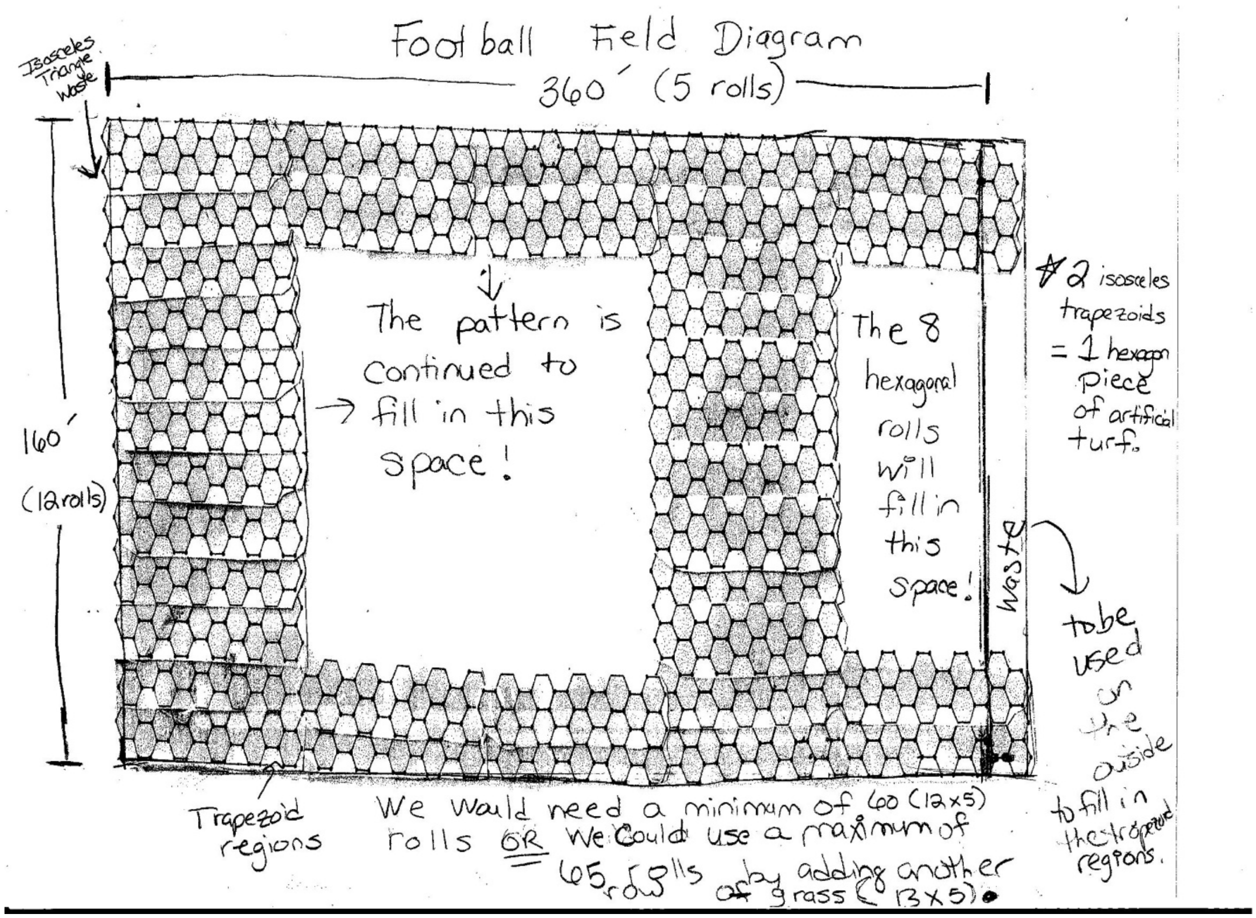
Keeping in mind that each side of each hexagonal piece is 5 feet, I then used my Geometry knowledge to recognize several of its properties. For one, each regular hexagon can be split into two isosceles trapezoids with one base being 5 feet and the other being 10 feet. I found this out because a regular hexagon can be split into 6 congruent equilateral triangles with sides 5 feet each, which makes the other base 10 feet. I then realized that this also makes the distance across (from end to end) one hexagon 10 feet. But, since the hexagons have to fit together to tessellate the plane, the distances would actually alternate between 10 feet and 5 feet, which makes each roll 75 feet across (since the roll is 10 hexagons long). Using my knowledge of apothems and 30-60-90 special right triangles, I found that the apothem for each hexagon is 2.5 * square root of 3, which makes each hexagon 5 * square root of 3 "wide". Since each cylindrical hexagonal roll is 2 hexagons wide, then each hexagon is 10 * square root of 3 wide.

Here are some of my illustrations:



Using this information, I estimated how many rolls I needed lengthwise and width wise by using the dimensions of the football field. At first, I thought that I would need 10 rolls widthwise because I took 160 feet and divided it by 10 * square root of 3. This gave me a decimal that was between 9 & 10. I also thought that I would need 5 rolls lengthwise because I took 360 feet and divided it by 75 and this gave me a decimal between 4 and 5. In other words, I thought that 50 rolls would be needed since it would be 10 x 5. But, once I started cutting the rolls out from my template and piecing them together, I realized that I had not considered the spaces that would be filled in by manipulating the rolls by “flipping them the opposite direction” multiple times. The diagram below shows how there needed to be a minimum of 12 rolls widthwise and 5 rolls lengthwise, for a minimum amount of 60 rolls. However, to save time, an additional row of artificial turf could be added to one end of the football field making it 13 rolls widthwise and 5 rolls lengthwise, for a combined total of 65 rolls. This is still better than the 116 cylindrical square rolls that would be needed to cover the same space at a much higher cost!

Here is another diagram showing how the football field would be covered along with information about how the extra pieces could be used to fill in the trapezoid pieces along the sides of the field:



Now, we are ready to share our cost results:

If Each Cylindrical Roll Costs $2400, How Much Would It Cost To Cover the Football Field With the Square Cylindrical Rolls?

$2400 * 116 = $278,400.

If Each Cylindrical Roll Costs $2400, How Much Would It Cost To Cover the Football Field With the Hexagonal Cylindrical Rolls?

$2400 * 60 = $144,000 (Cost for Student Option # 1 and Minimum Cost for Student Option # 2)

$2400 * 65 = $156,000 (Maximum Cost for Student Option #2)

What Is Our Recommendation to Mr. Walker, our School Athletic Director?

We recommend that you choose the hexagonal cylindrical rolls. By doing so, you would save lots of money for our school! In fact, the cost savings would be between $122,400 and $134,400 (Difference between the cost of the cylindrical square rolls and the cost of the cylindrical hexagonal rolls).

We hope that you consider all the facts that we have presented to you. We really enjoyed solving this real-world problem and we appreciated the chance to help you, as our Athletic Director, save our school money! Please let us know if you need any further clarification on our work.

Sincerely,

Sarah Adams & James Perdue

Ms. Shields' Geometry Class